

4.4 Polynomial Rates of Change

SWBAT explore how quickly polynomial functions increase and decrease.

In as many ways as possible, compare and contrast linear, quadratic, and exponential functions.

Answers will vary

Example 1: Use the following functions to help answer the statements below.

2^x	$x^2 - 20$	$x^5 - 4x^2 + 1$	$x + 30$	$x^4 - 1$	$x^3 + x^2 - 4$	$-x^2 + x$
-------	------------	------------------	----------	-----------	-----------------	------------

a) Write the following expressions in order from **least to greatest** when the value of x is zero.

$$\underbrace{(x^2 - 20)}_{-20} \quad \underbrace{(x^3 - x^2 - 4)}_{-4} \quad \underbrace{(x^4 - 1)}_{-1} \quad \underbrace{(-x^2 + 3x)}_0 \quad \underbrace{(2^x \text{ or } x^5 - 4x^2 + 1)}_1 \quad \underbrace{(x + 30)}_{30}$$

b) Do you think this order would change when x represents other numbers?

yes b/c the values all grow at different rates.

c) Write the same expressions from **least to greatest** when x represents a very large number (this number is so large it is close to or approaching positive infinity).

$$\text{Least} \quad \underbrace{(-x^2 + 3x)} \quad \underbrace{(x + 30)} \quad \underbrace{(x^2 - 20)} \quad \underbrace{(x^3 + x^2 - 4)} \quad \underbrace{(x^4 - 1)} \quad \underbrace{(x^5 - 4x^2 + 1)} \quad \underbrace{(2^x)}_{\text{Greatest}}$$

d) Write the same expressions in order from **least to greatest** when x represents a number that is approaching negative infinity.

$$\text{Least} \quad \underbrace{(x^5 - 4x^2 + 1)} \quad \underbrace{(x^3 + x^2 - 4)} \quad \underbrace{(-x^2 + x)} \quad \underbrace{(x + 30)} \quad \underbrace{(2^x)} \quad \underbrace{(x^2 - 20)} \quad \underbrace{(x^4 - 1)}_{\text{Greatest}}$$

e) When comparing expressions, how does the order change depending on the values of x (close to negative infinity, zero, and positive infinity)?

When x is a large #, functions are ordered based on exponents

When x is a small #, functions are ordered from largest odd to largest even.

f) Determine where you would insert the following expression in question c.

$\left(\frac{1}{2}\right)^x$	x^7	$-x^5$	x^6	x^5
------------------------------	-------	--------	-------	-------

$$\text{Least} \quad \underbrace{(-x^5)} \quad \underbrace{(-x^2 + 3x)} \quad \underbrace{\left(\frac{1}{2}\right)^x} \quad \underbrace{(x + 30)} \quad \underbrace{(x^2 - 20)} \quad \underbrace{(x^3 + x^2 - 4)} \quad \underbrace{(x^4 - 1)} \quad \underbrace{(x^5)} \quad \underbrace{(x^5 - 4x^2 + 1)} \quad \underbrace{(x^6)} \quad \underbrace{(x^7)} \quad \underbrace{(2^x)}_{\text{Greatest}}$$

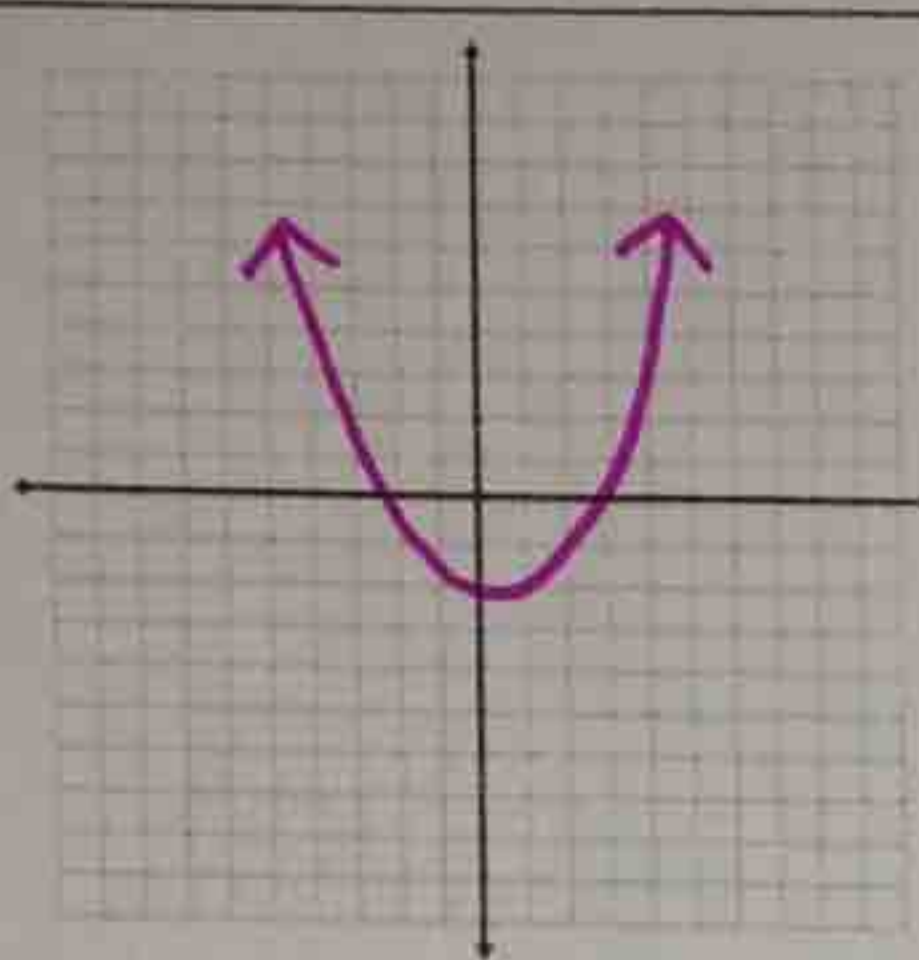
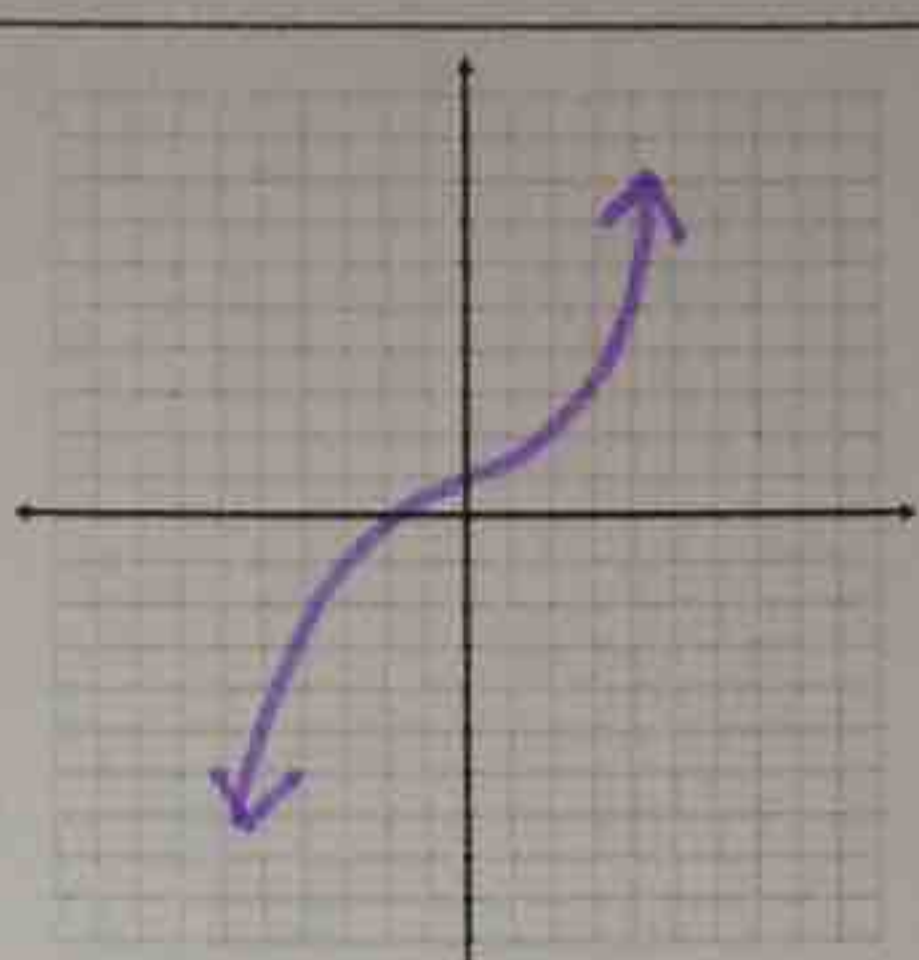
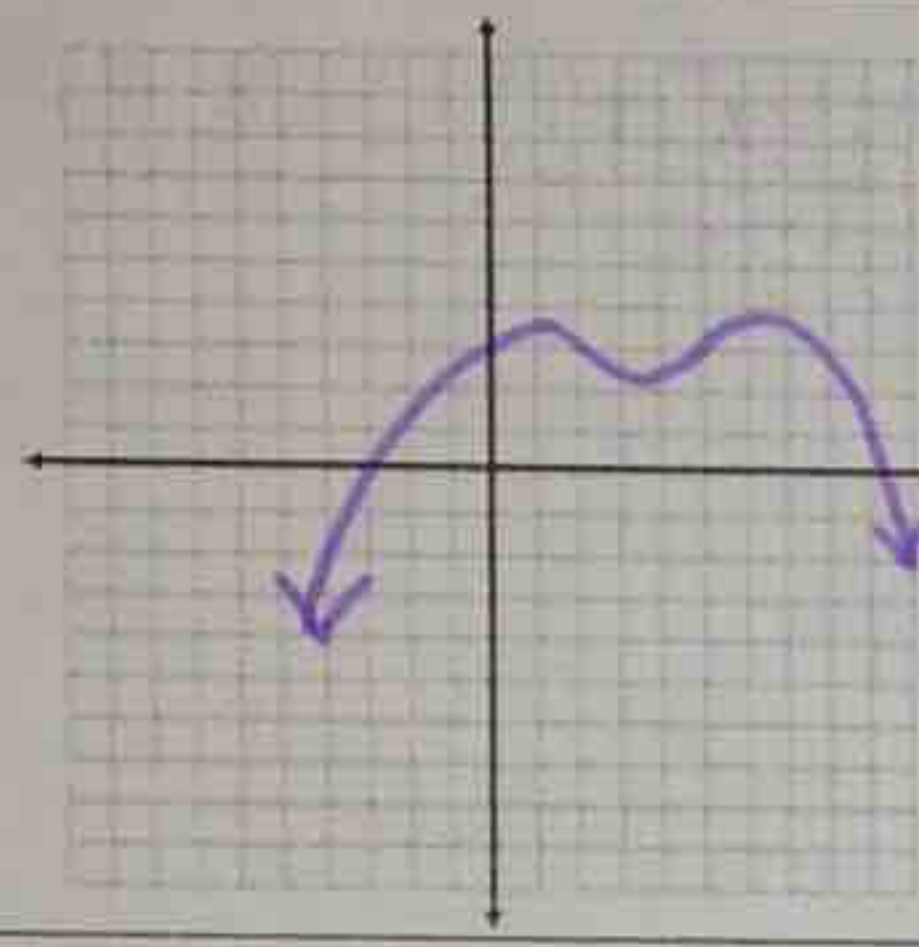
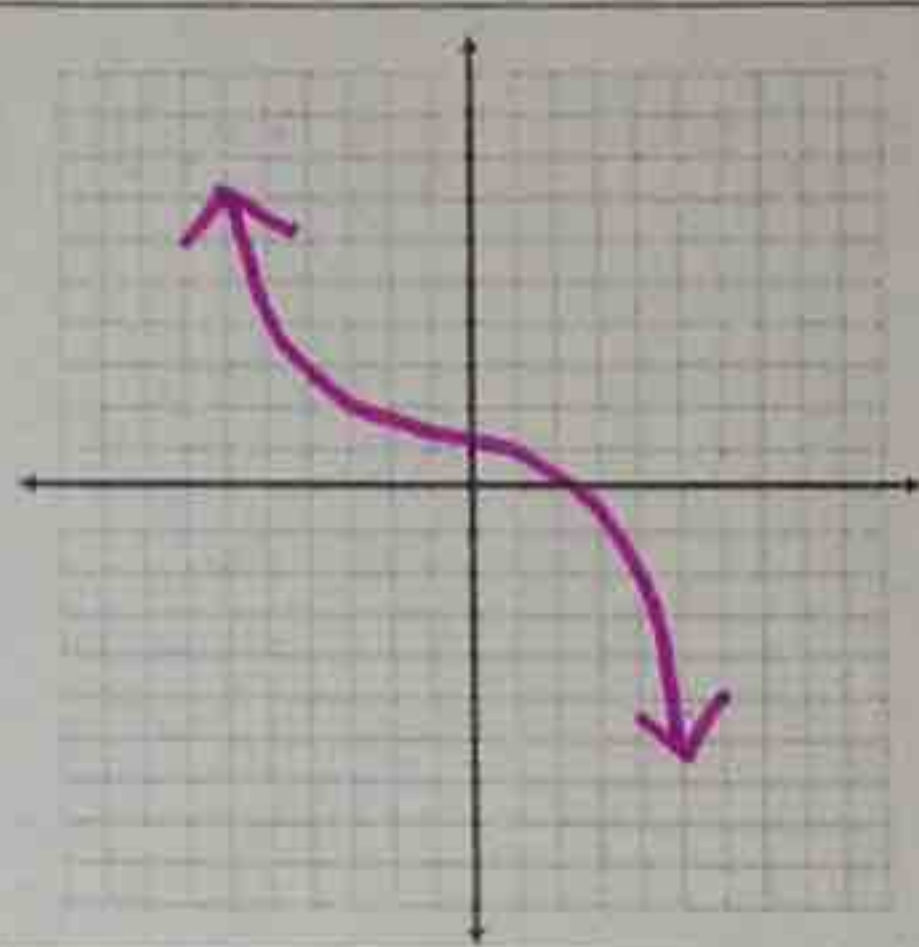
g) Now insert these same expressions into your list in question d.

$$\text{Least} \quad \underbrace{(x^7)} \quad \underbrace{(x^5 - 4x^2 + 1)} \quad \underbrace{(x^5)} \quad \underbrace{(x^3 + x^2 - 4)} \quad \underbrace{(-x^2 + x)} \quad \underbrace{(x + 30)} \quad \underbrace{(2^x)} \quad \underbrace{(x^2 - 20)} \quad \underbrace{(x^4 - 1)} \quad \underbrace{(-x^5)} \quad \underbrace{(x^6)} \quad \underbrace{\left(\frac{1}{2}\right)^x}_{\text{Greatest}}$$

h) Write your process for ordering one variable polynomial expressions for both extremes (when x approaches infinity as well as when x approaches negative infinity).

see question e.

End Behavior: Defined by what is going on at the ends of a graph.

	Even Degree Polynomial	Odd Degree Polynomial
Positive Leading Coefficient	 <p>EX: $3x^2$</p> <p>$x \rightarrow -\infty, f(x) \rightarrow \infty$</p> <p>$x \rightarrow \infty, f(x) \rightarrow \infty$</p>	 <p>EX: x^5</p> <p>$x \rightarrow -\infty, f(x) \rightarrow -\infty$</p> <p>$x \rightarrow \infty, f(x) \rightarrow \infty$</p>
Negative Leading Coefficient	 <p>EX: $-4x^6$</p> <p>$x \rightarrow -\infty, f(x) \rightarrow -\infty$</p> <p>$x \rightarrow \infty, f(x) \rightarrow -\infty$</p>	 <p>EX: $-2x^7$</p> <p>$x \rightarrow -\infty, f(x) \rightarrow \infty$</p> <p>$x \rightarrow \infty, f(x) \rightarrow -\infty$</p>

Example 1: Find the end behavior based on your knowledge of the function.

1. $f(x) = 3 + 2x$

Degree: 1

LC: 2

End Behavior:

$x \rightarrow -\infty, f(x) \rightarrow -\infty$

$x \rightarrow \infty, f(x) \rightarrow \infty$

2. $f(x) = x^4 - 16$

Degree: 4

LC: +1

End Behavior:

$x \rightarrow -\infty, f(x) \rightarrow \infty$

$x \rightarrow \infty, f(x) \rightarrow \infty$

3. $f(x) = 3^x$

Degree: exponential

LC: Growth

End Behavior:

$x \rightarrow -\infty, f(x) \rightarrow 0$

$x \rightarrow \infty, f(x) \rightarrow \infty$

4. $f(x) = x^3 + 2x^2 - x + 5$

Degree: 3

LC: +1

End Behavior:

$x \rightarrow -\infty, f(x) \rightarrow -\infty$

$x \rightarrow \infty, f(x) \rightarrow \infty$

5. $f(x) = -2x^3 + 2x^2 - x + 5$

Degree: 3

LC: -2

End Behavior:

$x \rightarrow -\infty, f(x) \rightarrow \infty$

$x \rightarrow \infty, f(x) \rightarrow -\infty$

6. $f(x) = \log_2 x$

Degree: Logarithm

LC: +1

End Behavior:

$x \rightarrow 0, f(x) \rightarrow -\infty$

$x \rightarrow \infty, f(x) \rightarrow \infty$

7. $f(x) = -2(x-3)(x+4)$

Degree: 2

LC: -2

End Behavior:

$x \rightarrow -\infty, f(x) \rightarrow -\infty$

$x \rightarrow \infty, f(x) \rightarrow -\infty$

8. $f(x) = \sqrt{x} - 3$

Degree: square root

LC: +1

End Behavior:

$x \rightarrow 0, f(x) \rightarrow -3$

$x \rightarrow \infty, f(x) \rightarrow \infty$

9. $f(x) = 3(x-1)(x+2)(x-4)$

Degree: 3

LC: 3

End Behavior:

$x \rightarrow -\infty, f(x) \rightarrow -\infty$

$x \rightarrow \infty, f(x) \rightarrow \infty$